Enroll No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous)

Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION NOVEMBER – 2016

M.Sc. Mathematics

16PMTCC03 - FUNCTIONS OF SEVERAL VARIABLES

Duration of Exam – 3 hrs ***Semester – I*** ***Max. Marks – 70***

***Part A** (5x2= 10 marks)*

Answer **ALL** questions

1. Define continuously differentiable function and directional derivative of function.
2. State Young’s and Schwarz theorem.
3. Define Euclidean Norm in R^n and find $||u||$ $when\ u = (-2, 3, 4)$.
4. Define Einstein’s summation convention and find the value of δ_i^i.
5. If a_{ij} are constant and $a_{ij} = a_{ji}$, calculate $\frac{\partial^2}{\partial x_k \partial x_l}(a_{ij}x_i x_j)$.

***Part B** (5X5 = 25 marks)*

Answer **ALL** questions

6a. If $f : R^n \to R$ is differentiable and if $f(0) = 0$ then prove that there exists $g_i : R^n \to R$ such that $f(x) = \sum_{i=1}^{n} x_i g_i(x)$.

OR

6b. Let $A \subset R^n$ be an open set, let $a \in A$ and let $f : A \to R$. If f has maximum value at point a and if $Df_i(a)$ exists then prove that $D_i f(a) = 0$.

7a. If $P : R^2 \to R$ is defined by $P(x, y) = x \bullet y$, then $DP(a,b)(x, y) = bx + ay$ also $P'(a,b) = (b,a)$. Then prove that P is differentiable.

OR

7b. Define Euclidean inner product. Let $x, y \in R^n$, then show that $|\|x\| - \|y\|| \leq \|x - y\|$.

8a. If $f : R^n \to R$ is a constant function then prove that $Df(a) = 0$.

OR

8b. Prove that Every Closed ball is closed set

9a. Define $f : R^2 \to R$ as $f(x) = \sin(x_1 x_2)$. Find $Df(a)$.

OR

9b. Let $x\ , y\ \in R^n\ and\ \alpha \in R$ then prove that $| < x, y > | = ||x|| * ||y||$ if and only if x and y are dependent.

10a. A function $f : R^n \to R$ is homogeneous of degree m if $f(tx) = t^m f(x), x \in R^n$ then show that $f(x)=\frac{1}{m}\sum_{i=1}^{n} x_i D_i f(x)$.

OR

10b. Define covariant and contravariant tensor of rank two. If x^i be the coordinate of a point in n-dimensional space show that dx^i are component of a contravariant vector.

<u>Part C</u> (5X7 = 35 marks)

Answer **<u>ALL</u>** questions

11a. Let $f,g:R^n \to R$ be differentiable at $a \in R^n$. Then

1) $D(f \pm g)(a) = Df(a) \pm Dg(a)$
2) $D(f.g)(a) = g(a).Df(a) + f(a)Dg(a)$
3) If $g(a) \neq 0$, then $D\left(\frac{f}{g}\right)(a) = \frac{g(a)Df(a) - f(a)Dg(a)}{[g(a)]^2}$

OR

11b. Prove by an example the fact $T: R^n \to R^m$ *and* $e_1, e_2, \ldots, e_n$ are standard basis in R^nthen $T((x_1, x_2, \ldots, x_n)) = T(e_1)x_1 + T(e_2)x_2 + \cdots + T(e_n)x_n$.

12a. State and prove chain rule.

OR

12b. Let $\mathrm{x, y} \in \mathrm{R^n}$ and $\alpha \in \mathrm{R}$. Then prove that

i) $||\mathrm{x}|| \geq 0$ and $||\mathrm{x}|| = 0$ ifand only if $\mathrm{x} = 0$.
ii) $|< x, y >| \leq ||\mathrm{x}|| ||\mathrm{y}||$
iii) $||\mathrm{x} + \mathrm{y}|| \leq ||\mathrm{x}|| + ||\mathrm{y}||$
iv) $||\alpha\mathrm{x}|| = \alpha||\mathrm{x}||$.

13a. Let $f: R^n \to R$ and let $a, x, y, e_i \in R^n$. Then

1) $De_i f(a) = D_i f(a)$
2) $Dsxf(a) = sD_x f(a)$
3) If f is differentiable at a, then $D_x f(a) = Df(a)(x)$

OR

13b. If $a_{ij}x^i x^j = 0$ where a_{ij} are constant then show that $a_{ij} + a_{ji} = 0$.

14a. Let $f: R^n \to R^m$ be differentiable at $a \in R^n$. Let $1 \leq i \leq n$ and let$1 \leq j \leq m$. Then the $(i,j)^{th}$ entry of the Jacobian matrix $f'(a)$ is exactly the i^{th} entry of the Jacobian matrix $f^{j'}(a)$.

OR

14b. Let $T: R^3 \to R^3$ be the transformation that sends each point to its orthogonal projection on $XZ - plane$. Show that T is linear transformation.

15a. If a function $f: R^n \to R$ is continuously differentiable at $a \in R^n$ then show that $Df(a)$ exists.

OR

15b. Define Kronecker delta. Show that $\frac{\partial \emptyset}{\partial x^i}$ is a covariant vectors of rank one where $\emptyset$ is a scalar function.